

<h1 style="margin: 0;">Non - independent system</h1>	<h1 style="margin: 0;">가</h1>
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Jain 1984
Jain
Jain

가 , 가

Jain 가가 ,

: ,

I.

, Jain

(fairness) 가

(resource) (capacity) Jain (metric)

(distributed systems) (qualitatively)

(fair allocation) . ([1]

가 .) , Jain

[1],[2],[3],[4],[5], [5] 가

[1] Jain (proper allocation metric)

(allocation)

90%

10% 가

(quantitatively) 가 (fairness index)

Jain [1],[2],[3] 가

Jain II

:
:

(fairness index)
 II
 Jain
 가 [4].

fairness index
 가 가
 가
 가
 Jain
 Jain
 (local fairness)
 (global fairness)

Jain
 가
 II Jain
 가 가
 III Jain
 가
 IV Jain

utilization

II. Jain fairness index

Jain i (normalized resource

allocation) x_i
 가 [1],[2],[3]. x_i
 throughput(y_i) throughput(z_i)
 ($x_i = y_i/z_i$) x_i 1.0
 가 [2],[3], x_i 가 가
 가 x_i
 가
 :

$$\text{fairness index} = \frac{(\sum x_i)^2}{n \times \sum x_i^2} \quad (1)$$

(1) n
 가
 가
 100% 1.0 x_i 가
 (\quad) x_i 가
 가 0.0
 (1)
 n^2

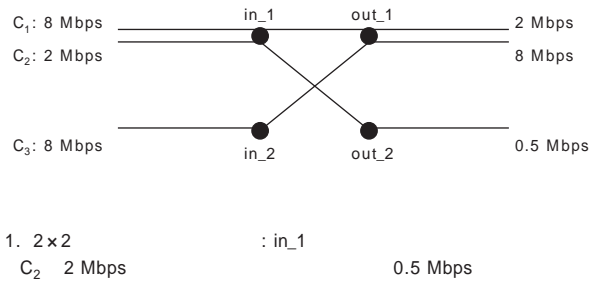
$$\begin{aligned} \text{fairness index} &= \frac{(\frac{1}{n} \sum x_i)^2}{\frac{1}{n} \times \sum x_i^2} = \frac{(E[X])^2}{E[X^2]} \\ &= \frac{(E[X])^2}{(E[X])^2 + \text{var}(X)} \end{aligned} \quad (2)$$

(2) $(E[X])^2$
 (coefficient of variance: C_X)

$$\text{fairness index} = \frac{1}{1 + \text{var}(X)/(E[X])^2} = \frac{1}{1 + C_X^2} \quad (3)$$

(2) $\text{var}(X)$ 가 X
 가 (3)
 C_X (unbounded) 가

가



(nonlinear reciprocal proportion) 가 (2) (3)

(resource) (capacity) (distributed systems)

(1) 가 Jain (discrimination) index (discrimination index) , Jain 가

Discrimination index = $1 - \text{fairness index} = 1 - \frac{(\sum x_i)^2}{n \times \sum x_i^2}$, Jain 가 [4].

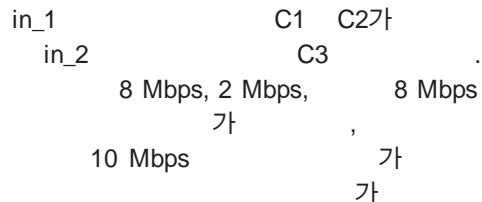
[1],[2],[3] Jain fairness index가 가

(utilization)

- Population size independency: 가
- Scale and metric independency:
- Boundedness: ,가 0 1
- Continuity: 가
- Intuitiveness: 가

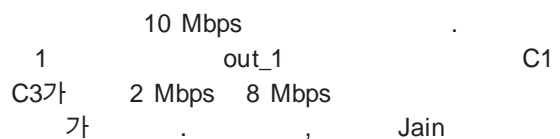
Fairness index 가

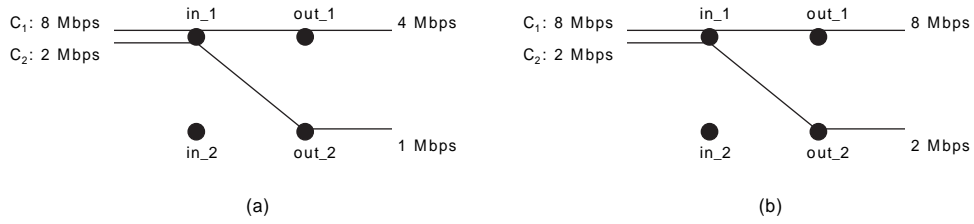
[1],[2],[3] fairness index 가



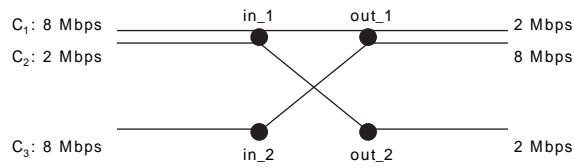
III. Jain

II Jain fairness index

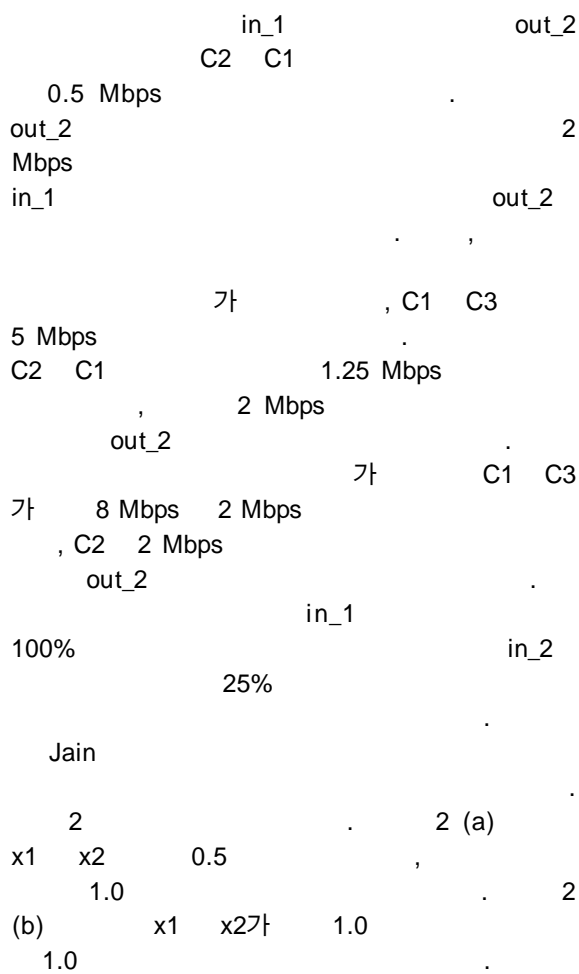




2.



3. 2x2

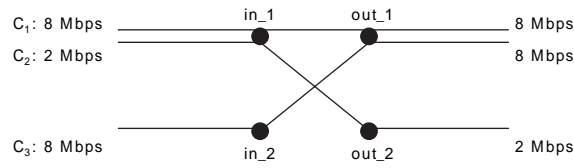


IV. Utilization - Wise Fairness Definition

III [1],[2],[3] Jain 가 utilization

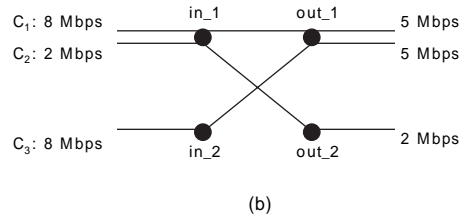
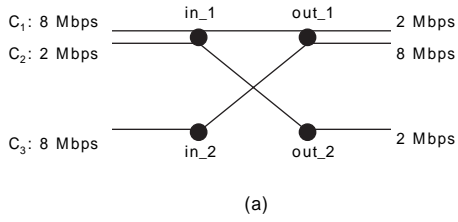
가 utilization

Jain 3



4. Internal speedup factor가 2

3 1 0.5 Mbps
 C2가 2 Mbps
 Jain
 x_1 x_2 0.25 1.0
 in_1 0.735가
 36.5% 8:2 C1 C3 2:8, 4:6, 5:5, 6:4
 out_1
 C1 C2가 2 Mbps
 (CIOQ: Combined Input
 -Output Queueing)
 internal speedup
 speedup
 factor 가
 , speedup factor 가
 , speedup factor가 2
 [8]가 가
 speedup factor가 2
 4 internal speedup
 factor가 2 가
 20 Mbps
 10 Mbps
 $x_1 = \frac{y_i}{\min(z_{i_in}, z_{i_out})}$ (4)
 가 (4) (1)
 , Jain 3
 x_1, x_2 10 Mbps
 $x_1 = \frac{2}{\min(8, 2)} = \frac{2}{2} = 1.0,$
 $x_2 = \frac{2}{\min(2, 10)} = \frac{2}{2} = 1.0,$
 in_1 4
 $x_1 = \frac{8}{\min(8, 10)} = \frac{8}{8} = 1.0,$
 $x_2 = \frac{2}{\min(2, 20)} = \frac{2}{2} = 1.0,$
 fairness index = $\frac{(2.0)^2}{2 \times (1.0 + 1.0)} = 1.0$
 1 C2
 가



5.

V.

IV utilization

$$x_i = \min(z_{i_1}, z_{i_2}, z_{i_3}, \dots, z_{i_k}) \quad (5)$$

$$\sum_{k=1}^j z_{i_j} \leq 0.0$$

overload [6]

overload

[6] Input-Queued

(MIQ: Multiple

(local fairness)

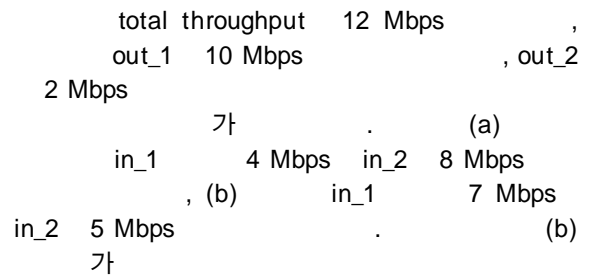
(global fairness)

가

V

가

5



$$x_i = \min \left(\sum_j y_{i,j}, \sum_j z_{i_{in}}, \sum_j z_{i_{k,out}} \right) \quad (6)$$

$$\sum_j y_{i,j} \leq x_i$$

$$(6) \quad (1) \quad 5$$

5 (a) :

$$x_1 = \min \left(\frac{2+2}{8+2}, \frac{4}{2+10} \right) = 0.4,$$

$$x_2 = \frac{8}{\min(10, 8)} = \frac{8}{8} = 1.0,$$

$$\text{fairness index} = \frac{(1.4)^2}{2 \times (0.16 + 1.0)} = 0.845$$

5 (b) :

$$x_1 = \min\left(\frac{5+2}{8+2}, \frac{7}{5+10}\right) = 0.7,$$

$$x_2 = \frac{5}{\min(10, 5)} = \frac{5}{5} = 1.0,$$

$$\text{fairness index} = \frac{(1.7)^2}{2 \times (0.49 + 1.0)} = 0.97$$

“ 가 가

가

5 (b)

(6) (5)

가

VI.

가

가,

(utilization)

가

가

가

가

Jain

가

가

Jain

가

가

x_i

가

[]

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Fairness Evaluation in the Non-Independent Systems

Fairness is one of the important considerations in the performance study of shared systems. To date, fairness of a

shared system has been evaluated quantitatively by using the well-known fairness index defined by Jain in 1984. However, the fairness index defined by Jain can be applied limitedly to the fairness evaluation of a single and independent shared system. When we try to apply Jain's fairness index to a non-independent shared system where sub-systems have inter-dependency, it causes a problem of underutilization of the system resources. In this paper, therefore, we newly define a fairness index based on the Jain's approach so as to evaluate the fairness of the non-independent shared system such as input-queueing switch. The fairness index defined in this paper preserves all the characteristics of Jain's fairness definition and makes it possible to efficiently use the system resource.

Keywords : fairness, fairness index



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