Throughput Analysis of the Bifurcated Input-Queued Packet Switches with Restricted Contention

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There suggested is an input-queueing scheme in which a buffer is divided into several smaller buffer blocks for enhancement of the limited throughput of the ordinary input-queued switch. The scheme is analysed and compared with respect to the performance of the scheme with free contention.

Bifurcated input-queued switches: For independent and uniform traffic, the throughput of an input-queued $N \times N$ packet switch is asymptotically upper bounded by 0.586 as $N \to \infty$ [1], while the output-queued switch can attain 100% throughput. Therefore, several approaches have been proposed for enhancement of the limited throughput of the traditional input-queued switch by the *Head-Of-Line* (HOL) blocking. Even though those approaches could achieve sizeable improvement in the throughput, they also resulted in such internal speedup as in the output-queued switch.

One successful idea for input-queued switches is a bifurcated queueing approach which divides the buffer for an input port into m smaller bifurcated buffers ($2 \leq m \leq N$). The *bifurcated buffer* originally means, as the name implies, the buffers divided into two. The term, however, can be used as the extension when the number of buffers is larger than two [2]. Defining an *arbitration round* as an arbitration for a given output port in a time slot, there are N arbitration rounds in an $N \times N$ switch. Then, in the bifurcated input-queued switch, an input port can attend m arbitration rounds in a time slot. If an input port is allowed to attend all m arbitration rounds we call this kind of contention and arbitration rule *free contention* and *multi-chance service rule*, respectively. The problem encountered with this approach is the internal speedup in memory access time occurred when an input port is selected multiple times for different outputs in a time slot. The throughput analysis for this kind of switch is found in [2].

In this Letter we propose a bifurcated input-queued packet switch with restricted contention. If an input port, selected in one of previous arbitration rounds to send a packet, is restricted in attending following rounds, we call these *restricted contention* and *single-chance service* rule. In this case the number of buffers within an input port is equal to the switch size ($m = N$) but there is no internal speedup since an input port is restricted not to send more packets than one. A model for this kind of switches is shown in Figure 1 and the throughput analysis follows in next section.

Throughput analysis: We assume that a packet arrives at each input port with probability λ per time slot, destined with equal probability ($1/N$) to any one of the output ports. Or, in terms of the bifurcated switches, we can say that in an input port a packet arrives at each bifurcated buffer for output j ($1 \leq j \leq N$) with probability $\lambda_j = \lambda/N$ per time slot, where N is the number of buffers within an input port. Packets in the buffers are served on an FCFS basis at the beginning of each time slot. In the case of the bifurcated input-queued switch with restricted contention, maximum N buffers, one buffer per input port, can attend an arbitration round during a slot since the input ports selected in previous arbitration rounds can not attend following rounds during the same slot. It implies that the throughput T_j for output j varies with the order of the arbitration round which the output port is coupled with in a slot. That is, the later the arbitration round, the smaller the throughput of the round in a slot since the number of input ports restricted in attending the arbitration increases. Therefore, we can obtain the average throughput per port by taking the average of the throughputs for N arbitration rounds during a time slot.

By the virtue of the uniformity the throughput for an output

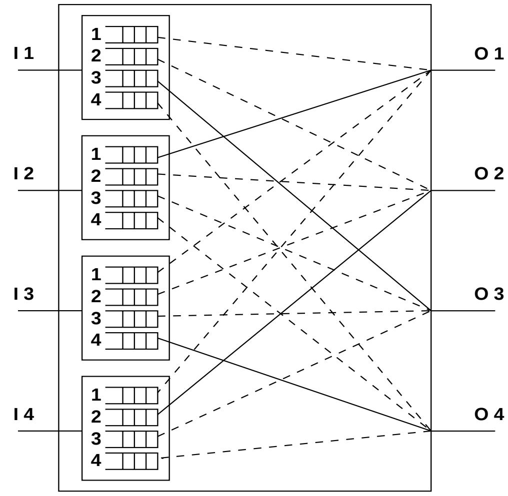

Figure 1: *A 4×4 bifurcated input-queued switch with restricted contention and single-chance service rule.*

port is statistically equal to the throughput of another port, and we use the average throughput T as a quantitative measure. Let $T_{j,k}$ be the throughput for output j when the related arbitration is performed in the kth order of arbitration round during a time slot. Let N_j be the number of HOL packets in the queues for output j at the boundary of a time slot and $N_{j,k}$ be N_j which can attend the kth arbitration round. By the restricted contention rule, $N_{j,k}$ generally decreases as the order (k) of arbitration round comes late and is less than or equal to N_j. If $N_{j,k} \geq 1$, one of $N_{j,k}$ packets must be serviced after the related arbitration round. This idea can be expressed easily with the indication function $\epsilon(x)$, $\epsilon(x) = 1$ for $x \geq 1$ and $\epsilon(x) = 0$ for $x = 0$. The throughput for the kth arbitration round is defined as the steady-state expected value

$$T_{j,k} = E[\epsilon(N_{j,k})]. \tag{1}$$

Let $N_{j,k}^{(b)}$ be the number of blocked HOL packets at the queues for output j after the kth arbitration round. Then, $N_{j,k}^{(b)} = N_{j,k} - \epsilon(N_{j,k})$ Taking expectation on both sides of the previous equation and combining it with (1), then we get

$$T_{j,k} = E[N_{j,k}] - E[N_{j,k}^{(b)}]. \tag{2}$$

We can get $T_{j,k}$ by deriving $E[N_{j,k}]$ and $E[N_{j,k}^{(b)}]$ of (2) in terms of $\lambda_{j,k}$ respectively since we have $T_{j,k} = \lambda_{j,k}$ in steady state where $\lambda_{j,k}$ is λ_j considered at the kth round.

(i) *derivation of* $E[N_{j,k}^{(b)}]$:
Let $R_{j,k}$ denote the number of released (namely, unblocked) HOL packets destining for output port j at the end of the kth arbitration round. Then,

$$R_{j,k} = N - (k-1) - N_{j,k}^{(b)}, \tag{3}$$

where $(k-1)$ indicates the number of queues for output j restricted at the kth arbitration round by the restricted contention rule. Let $\rho_{j,k}$ denote the steady-state probability considered at the kth round that a queue for output j has a fresh HOL packet given that the queue is not blocked during the previous time slot. Since every packet will eventually become a fresh HOL packet at some point, the following flow conservation relationship must hold. Thus, $E[R_{j,k}]\rho_{j,k} = \lambda_{j,k}$. That is, the arrival rate of new packets for a given output port at the kth round is equal to the average number of HOL packets for the output at the same round. This equation can be thought as the statistical limitation of $E[R_j]\rho_j = \lambda_j$ to the kth round. By taking expectations on both sides in (3)

$$E[N_{j,k}^{(b)}] = N - (k-1) - \frac{\lambda_{j,k}}{\rho_{j,k}}.$$

(ii) *derivation of* $E[N_{j,k}]$:
Let N_j' be the value of N_j for the next time slot, given by

$$N_j' = N_j - \epsilon(N_j) + A_j, \qquad (4)$$

where A_j is the number of fresh HOL arrivals at the released HOL positions (R_j) and is independent of N_j for large N (the number of buffers within an input port). Taking expectation on both sides of (4) and considering the steady state, we get $E[\epsilon(N_j)] = E[A_j]$ since $E[N_j'] = E[N_j]$ in the steady state. In order to find $E[N_{j,k}]$, considering the kth order in a time slot after squaring and taking expectation on (4),

$$E[N_{j,k}] = \lambda_{j,k} + \frac{(\lambda_{j,k})^2}{2(1-\lambda_{j,k})},$$

since $E[A_j(A_j - 1)] \approx (\lambda_j)^2$ as in [3].

(iii) *Average throughput per port* (T):
Based on derivations of (i) and (ii) and letting $\rho_{j,k} = 1$ since we have interest in maximizing throughput, we can get from (2)

$$\lambda_{j,k} = N - (k-1) + 1 - \sqrt{(N-(k-1))^2 + 1}.$$

Consequently, the average throughput per port is

$$T = \frac{1}{N}\sum_{k=1}^{N} \lambda_{j,k}. \qquad (5)$$

In the above analysis, note that N means the number of buffers within an input port rather than the switch size, even though the two are equal for the bifurcated input-queued switch with restricted contention as mentioned previously. The closed formula (5) gives very high throughput per port (≈ 1.0) as $N \to \infty$. For example, $T = 0.975$ for $N = 100$ which is a sizeable enhancement of throughput.

It is noteworthy that the removal of the term $(k-1)$ in above derivation, which means the number of restricted queues in the kth arbitration round, results in $T = N + 1 - \sqrt{N^2+1}$ which corresponds to the throughput of the bifurcated input-queued switch with free contention as in [2]. In this case, the throughput also comes to 1.0 as $N \to \infty$, which is slightly higher than that of the restricted contention case.

Conclusions: In this Letter, we have proposed and analysed the bifurcated input-queued packet switch with restricted contention. The switch provides the throughput close to 1.0 since N times arbitration rounds are performed in a time slot, while it does not require any internal speedup since an input is restricted to send at most one packet in a time slot. The analysis result can be extendible to the bifurcated input-queued switch with free contention and to the ordinary input-queued switch.

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