Derivation of the mean cell delay and cell loss probability for multiple input-queued switches

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Abstract— **The multiple input-queued (MIQ) ATM switch has drawn much interest recently as a promising candidate for a high-speed and high-performance packet switch. The most conspicuous feature of the switch is that each input port is equipped with m ($1 \leq m \leq N$) distinct queues, each for a group of output ports. Since the MIQ switch has multiple queues, an input can serve up to m cells in a time slot, leading to an enhanced performance. In this paper, we derive the average queue length, mean cell delay, and cell loss probability for the MIQ switch in terms of the number of queues in an input port (m) and input load. The results include a special case of the single input-queued (SIQ) switch ($m = 1$), which is analyzed by Hui *et al.* [1].**

Keywords— **Multiple input queueing, ATM switch**

I. Introduction

To IMPROVE the limited throughput $(2-\sqrt{2})$ of the nonblocking ATM switch with a single FIFO in an input port, multiple input-queued (MIQ) switches have been proposed [2], [3]. In the switch, each input is equipped with m $(1 \leq m \leq N)$ separate FIFOs as shown in Fig. 1, where N is the switch size, and each FIFO is dedicated generally to an output group including N/m output ports. The queues in an input port and the number of queues (m) are referred to as *the bifurcated queue* and to as *the bifurcation parameter*, respectively, since the number of queues usually takes on a 2's power [3], [4]. With the multiple input-queueing approach, it is proved analytically that, when the bifurcation parameter m is large enough, 100% throughput can be attainable for homogeneous arrival traffics without either internal speedup or expansion of the switch fabric [2], [4], [5]. However, the average queue length, mean cell delay, and cell loss probability have still been evaluated by the computer simulation, not by numerical analyses.

In this paper, we mathematically analyze the average queue length, mean cell delay, and cell loss probability for the MIQ switch after describing the queueing model for the switch. We further compare them with the results for the SIQ switch in [1].

II. Queueing model and analysis

Before modeling the queue of the MIQ switch, we first assume that time is slotted and each slot carries an ATM cell. Cells are assumed to arrive at input ports or queues just after the time boundaries and depart from input ports or queues just before the succeeding time boundaries; the early-arrival model is assumed [6]. We further assume that the arrival traffics at each input are distributed independently and identically and the arrived traffics are distributed uniformly for all output ports, i.e., the homogeneous Bernoulli process. Then, since the bifurcated queues are in effect identical to one another under the homogeneity assumption of the arrival traffic, we shall consider only one queue.

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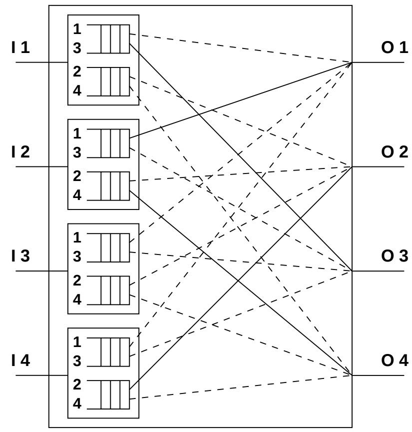

Fig. 1. Multiple input-queued (MIQ) switch ($N = 4$ and $m = 2$).

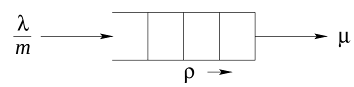

Fig. 2. $Geo/Geo/1$ queueing model.

Let λ and μ designate the mean arrival rate and the mean service rate for an input port, respectively. Since the arrival traffic is distributed uniformly among m queues in the same input, the mean arrival rate for a queue becomes λ/m. On the other hand, the mean service rate for a queue is still μ, since a queue is able to attend output contentions m times in a time slot and is selected with the probability of $1/m$ for each contention. Usually, the queue used in output-queued ATM switches is modeled as a $Geo/D/1$ queue since the ATM cell has the fixed cell size of 53 bytes. It is noteworthy that, in references [7] and [8], the authors adopted an $M/D/1$ queueing model for the analysis of the SIQ switch and used a discrete-time $Geo/G/1$ model to determine an exact formula for the expected waiting time of the SIQ switch. In the input-queued switch, however, the inter-service time between cells served consecutively has a geometric distribution due to the output conflict. even though it takes a fixed time to serve a cell itself. Therefore, we rather model the queue with $Geo/Geo/1$ model as shown in Fig. 2 than $Geo/D/1$ model.

For the $Geo/Geo/1$ queueing model, let ρ specify the steady-state probability that a queue has a fresh HOL cell which is just moved to the HOL position, given that the queue is not blocked during the previous slot. Then, we can use the relationship between ρ and λ for the same queueing model as provided in [2]:

$$(2-\rho)\lambda^2 - 2(1+m\rho)\lambda + 2m\rho = 0\,. \qquad (1)$$

Now, the value of μ as a function of λ can be computed as follows: The mean number of slots before a fresh HOL cell is served is $1/\mu$ since the number of slots is distributed according to the geometric distribution. After a cell is served, the mean number of slots before an arrival of a fresh HOL cell is $(1-\rho)/\rho$ since the number of slots is distributed according to the modified geometric distribution. In steady state, since the sum of these two terms is equal to the interarrival time of cells at an input port, m/λ, we have immediately

$$\frac{1}{\mu} + \frac{1-\rho}{\rho} = \frac{m}{\lambda}\,.$$

This relationship, together with the relation of ρ and λ in (1), gives

$$\mu = \frac{2(1-\lambda)}{2-\lambda}\,. \qquad (2)$$

Here, let the random variable K be the number of cells in a queue just before the arbitration phase. We shall consider a queue of infinite size for a moment. Then, the queue length K' for the next slot is modeled by

$$K' = K + \alpha - \gamma\epsilon(K) \qquad (3)$$

in which α is a Bernoulli random variable with $E[\alpha] = \lambda/m$, γ is also a Bernoulli random variable with $E[\gamma] = \mu$, the probability that the HOL cell is served. The indication function $\epsilon(x)$ is equal to 1 for $x \geq 1$ and 0 for $x = 0$.

From (3), the probability generating function $G(z)$ for the steady-state probability $p_k = \Pr[K = k]$ is given by

$$G(z) = E[z^{\alpha}]E[z^{K-\gamma\epsilon(K)}] \qquad (4)$$

since $K' = K$ in steady state. The first expectation of (4) is equal to

$$E[z^{\alpha}] = [1 - \lambda/m + (\lambda/m)z] \qquad (5)$$

and the second expectation becomes

$$\begin{aligned} E_{\gamma}\left[\sum_{k=0}^{\infty} p_k z^{k-\gamma\epsilon(k)}\right] &= p_0 + E_{\gamma}\left[\sum_{k=1}^{\infty} p_k z^{k-\gamma}\right] \\ &= p_0\left[1 - E_{\gamma}[z^{-\gamma}]\right] + E_{\gamma}\left[\sum_{k=0}^{\infty} p_k z^{k-\gamma}\right] \\ &= p_0\left[1 - E_{\gamma}[z^{-\gamma}]\right] + G(z)E_{\gamma}[z^{-\gamma}] \qquad (6) \end{aligned}$$

where $E_{\gamma}[\cdot]$ is the expectation for the random variable γ. Substituting (5), (6), and $E_{\gamma}[z^{-\gamma}] = [1 - \mu + \mu z^{-1}]$ into (4), we have

$$\begin{aligned} G(z) &= \frac{m\mu p_0(z-1)(m-\lambda+\lambda z)}{(\mu-1)\lambda z^2 + \big((m-\lambda)\mu - (1-\mu)\lambda\big)z - (m-\lambda)\mu} \\ &= \frac{m\mu p_0(m-\lambda+\lambda z)}{(m-\lambda)\mu - (1-\mu)\lambda z}\,. \qquad (7) \end{aligned}$$

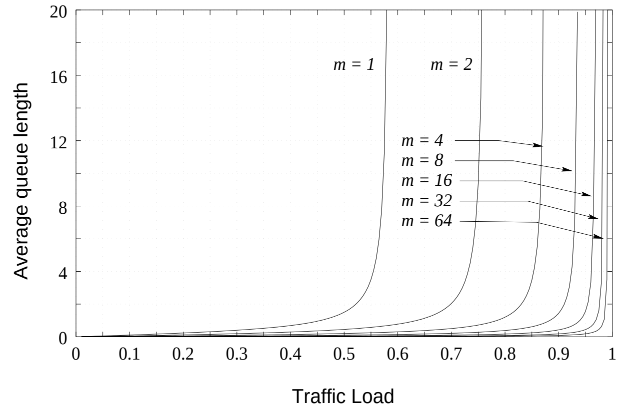

Fig. 3. Average queue length ($E[K]$) of the MIQ switch.

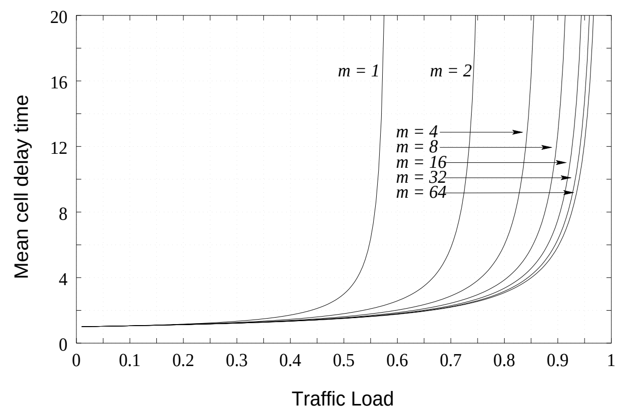

Fig. 4. Mean cell delay time of the MIQ switch.

After evaluating p_0 by setting $G(1) = 1$, and substituting p_0 in (7), we obtain

$$G(z) = \left(\mu - \frac{\lambda}{m}\right)\frac{(m-\lambda+\lambda z)}{(m-\lambda)\mu - (1-\mu)\lambda z}\,. \qquad (8)$$

Differentiating $G(z)$ with respect to z and setting $z = 1$, then the expected number of cells in a queue, $E[K]$, corresponds to the average queue length. The average queue length is given by

$$E[K] = \frac{\lambda}{m}\frac{m-\lambda}{m\mu-\lambda}\,. \qquad (9)$$

Substituting (2) into (9), then the expected value of K becomes

$$E[K] = \frac{\lambda}{m}\frac{(m-\lambda)(2-\lambda)}{2m - 2(m+1)\lambda + \lambda^2}\,. \qquad (10)$$

The mean cell delay, D, is obtained by the Little's theorem. That is,

$$D = \frac{E[K]}{\lambda/m} = \frac{(m-\lambda)(2-\lambda)}{2m - 2(m+1)\lambda + \lambda^2}\,.$$

Fig. 3 and Fig. 4 plot the average queue length and mean cell delay of the MIQ switch as a function of input load (λ) and the bifurcation parameter (m), respectively. As shown in the figures, the average queue length and mean cell delay become very small even under the high offered load, as the number of queues in an input (m) increases. It is intuitively clear that the average queue length becomes negligibly small when the number of queues in an input port (m) is large enough even at a high traffic load, since the arrived cells are stored at one of m queues. On the other hand, the mean cell delay has moderate values since the stored cells are selected with the probability of $1/m$. It is noteworthy from Fig. 3 and Fig. 4 that curves when $m = 1$ is exactly equal to the results for the SIQ switch in [1].

Besides the analysis of the average queue length and mean cell delay, the cell loss probability is another important performance measure for ATM switches. To the end, we need to evaluate the steady-state probability p_k. Let's expand (8) further into a series expansion form as follows:

$$\begin{aligned} G(z) &= \frac{1}{m}(1-\omega_\lambda)\frac{m-\lambda+\lambda z}{1-\omega_\lambda z} \\ &= \frac{1}{m}(1-\omega_\lambda)(m-\lambda+\lambda z)\sum_{k=0}^{\infty}\omega_\lambda^k z^k\,, \end{aligned} \tag{11}$$

where $\omega_\lambda = (\lambda(1-\mu)/(m-\lambda)\mu) = \lambda^2/(m-\lambda)(1-\lambda)$. Then, from the definition of $G(z)$, $G(z) = \sum p_k z^k$, we have

$$\begin{aligned} p_0 &= (1-\omega_\lambda)\left(1-\frac{\lambda}{m}\right) \\ p_k &= (1-\omega_\lambda)\left[\left(1-\frac{\lambda}{m}\right)\omega_\lambda + \frac{\lambda}{m}\right]\omega_\lambda^{k-1} \\ &= \left(\frac{\lambda}{m(1-\lambda)}\right)(1-\omega_\lambda)\omega_\lambda^{k-1}\,; \qquad k \geq 1\,. \end{aligned}$$

The cell loss probability (CLP), which is the buffer overflow probability for a finite buffer of size B, is upper bounded by the probability of $K > B$ for the case of infinite buffer size. Therefore,

$$\begin{aligned} CLP < \Pr[K > B] &= \left[\left(1-\frac{\lambda}{m}\right)\omega_\lambda + \frac{\lambda}{m}\right]\omega_\lambda^B \\ &= \frac{\lambda}{m(1-\lambda)}\omega_\lambda^B\,. \end{aligned}$$

Fig. 5 plots the upper bound of the cell loss probability over different values of m when the queue size (B) is 16 cells. The cell loss probability is also enhanced considerably as m increases.

III. Conclusions and remarks

In this paper we analyzed the average queue length, mean cell delay, and cell loss probability for the MIQ switch as functions of the number of queues in an input port and input load. As plotted in Figs. 3 through 5, the performance measures are much enhanced as the number of queues increases. Noting that the average queue length is about one mth of the mean cell delay and that the total required memory space for the MIQ switch is m times the

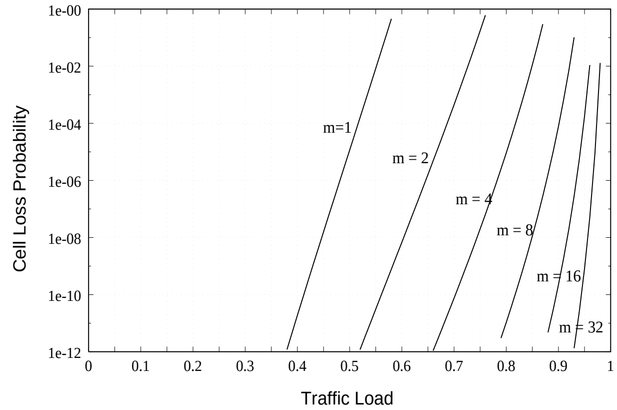

Fig. 5. Cell loss probability of the MIQ switch. ($B = 16$)

average queue length, we find that the MIQ switch requires almost constant memory space irrespective of the value of m. This further means that the MIQ switch requires no additional memory more than that for the SIQ switch. However, the multiple input-queueing approach could require either the internal speedup or the expansion in the switch fabric, and output buffers, when an input port switches multiple cells in a time slot. Therefore, we are requested to devise a simple and intelligent cell scheduling scheme which finds out a maximal input-output pairs under the constraint that each input is matched to at most one output and each output to at most one input. With a proper scheduling scheme, the MIQ switching scheme could be a high-speed and high-performance ATM switch alternative.

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